



Numerical simulation of astrophysical plasma with relativistic equation of state

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Abstract. In this paper we incorporate the relativistic equation of state of multi-species fluid in relativistic hydrodynamic code. We show that in the trans-relativistic temperature regime, composition plays a very important role in the temporal evolution of relativistic hydrodynamics. We show this for shock tube tests, as well as, relativistic jets.

Keywords : hydrodynamics — methods: numerical-relativity

1. Relativistic codes and equation of state of fluid

Relativistic upwind numerical codes have been built based on various schemes, *e.g.*, van Leer scheme (Falle & Komissarov 1996), piecewise parabolic method or PPM (Marti & Muller 1996; Mignone, Plewa & Bodo 2005), Godunov scheme (Sokolov, Zhang & Sakai 2001), TVD scheme (Ryu, Chattopadhyay & Choi 2006; hereafter RCC06) etc. A fluid is called thermally relativistic if $kT/mc^2 \gtrsim 1$, where k , T , m are the Boltzmann constant, temperature, and the rest mass of the gas particles, respectively. An equation of state (EoS) of a fluid composed of relativistic particles called RP (relativistic perfect), was calculated long ago (Chandrasekhar 1938; Synge 1957). The numerical codes based on RP EoS were found to be computationally expensive (Falle & Komissarov 1996). Approximations of RP EoS were proposed (Mathews 1971, RCC06) and codes based on such EoS were developed (Mignone et al. 2005, RCC06, Ryu, Chattopadhyay & Jang 2011). Since the thermal state of a fluid depends on the ratio T/m , composition may play a crucial role. Analytical solutions of blackhole accretion showed strong dependence on composition (Chattopadhyay 2008; hereafter C08, Chattopadhyay & Ryu 2009; hereafter CR09, Chattopadhyay & Chakrabarti 2011; hereafter CC11, Chattopadhyay et al. 2012). In this paper, we incorporate the EoS for multi-species fluid into the relativistic TVD code.

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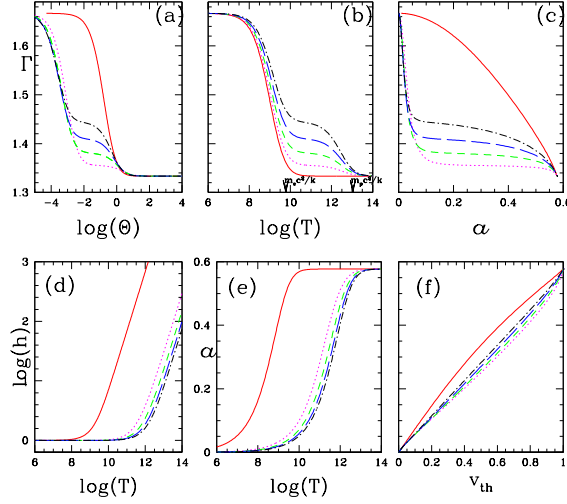


Figure 1. Γ as a function of (a) $\log(\Theta)$; (b) $\log(T)$; and (c) a ; (d) enthalpy as a function of T in log-log scale; (e) a as a function of $\log(T)$ and (f) a as a function of v_{th} . Each curve is characterized by $\xi = 0$ (solid), $\xi = 0.25$ (dotted), $\xi = 0.5$ (dashed), $\xi = 0.75$ (long-dashed), and $\xi = 1$ (dashed-dotted).

In the relativistic regime, the equations of motion in compact form, are

$$q_i + f_{,i} = 0; \text{ where } q \equiv (D, M_i, E)^T; \text{ and } f \equiv (Dv_j, M_iv_j + p\delta_{ij}, E + p)^T, \quad (1)$$

where, $D = \gamma\rho$, $M_i = \gamma^2\rho hv_i$, $E = \gamma^2\rho h$. Here, $\gamma = 1/(1 - v_i^2)^{1/2}$ is the Lorentz factor, where v_i s are the components of 3-velocity. A relation between the mass-density ρ , pressure p , and enthalpy h , is the EoS, and for multi-species fluid the EoS is (CR09),

$$h = 1 + \frac{\Theta}{\kappa} \left[\kappa_1 \frac{9\Theta + 3\kappa}{3\Theta + 2\kappa} + \eta\kappa_2 \frac{9\eta\Theta + 3\kappa}{3\eta\Theta + 2\kappa} + \kappa \right], \quad (2)$$

where, $\Theta = p/(\rho c^2)$, $\kappa = 1 - \xi(1/\eta - 1)/(2 - \xi + \xi/\eta)$, $\kappa_1 = (\kappa - \eta)/(1 - \eta)$, $\kappa_2 = (1 - \kappa)/(1 - \eta)$, and $\eta = m_e/m_p$ the ratio of electron to proton mass. Sound speed is $a = \Gamma p/(\rho h)$, $\Gamma = 1 + 1/N$, where, $N = \partial h/\partial \Theta - 1$. The thermal speed v_{th} can also be estimated since $e = \rho h - p = \rho c^2/(1 - v_{th}^2)^{1/2}$. The composition parameter is $\xi = n_{p^+}/n_{e^-}$ the ratio of proton and electron number densities. Equations 1 and 2 are solved by obtaining the complete eigen structure, and then using the eigen values and eigen vectors to reconstruct the fluxes (see RCC06 for details). In figure 1a-f, the thermodynamic properties like Γ , h , a and v_{th} are plotted to show their dependence on the composition parameter ξ . At same T (C08, CR09, CC11) pair-plasma ($\xi = 0$) is more relativistic compared to plasma containing protons, but at the same Θ , plasma with $\xi = 0.25$ is more relativistic (i.e., Γ is closer to $4/3$) than either $\xi = 0$ or electron-proton ($\xi = 1$) fluid. The same is true when fluids with different ξ are compared at the same a , a case which arises while comparing fluid which starts with the same Mach number or, in case of accretion disc fluid (see, C08, CR09, CC11).

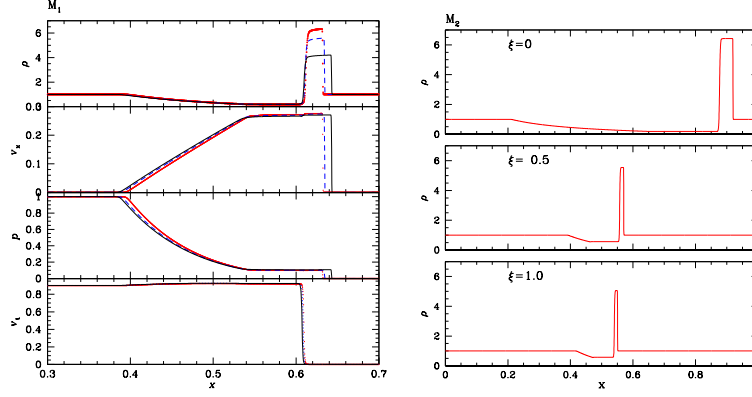


Figure 2. Shock tube test M_1 : Initial states: $\rho_L = \rho_R = 1$, $p_L = 1$, $p_R = 10^{-5}$, $v_{xL} = v_{xR} = 0$, and $v_{tL} = 0.9$, $v_{tR} = 0$. We plot ρ , v_x , p and v_t as a function of x at time $t = 0.4$. The plots are for $\xi = 0$ (solid), $\xi = 1$ (dashed) and $\xi = 0.2$ (solid points). M_2 : ρ vs x at $t = 0.5$ for fluids of three ξ . All fluids start with same temperatures on either side $T_L = 10^{11}$ K and $T_R = 10^5$ K.

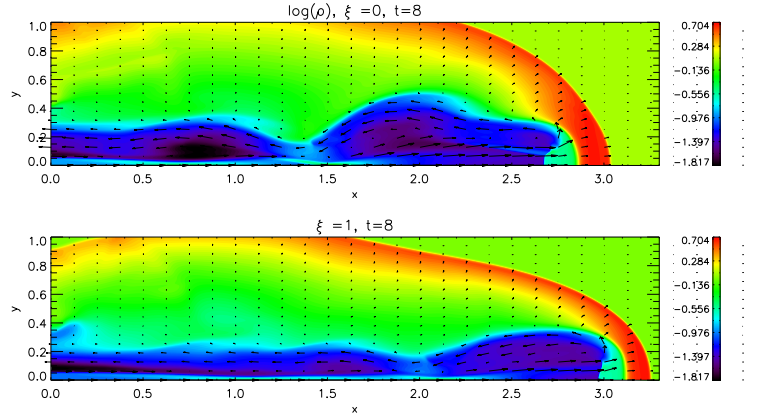


Figure 3. Density contours (log scale) and velocity vectors of 2D slab jet at time $t = 0.8$. Initial Conditions: $v_b = 0.9$, $\rho_b = 0.1$, $a_b = 0.1156$, i.e., $M_b = 7.785$, and $\rho_a/\rho_b = 10$ and $a_a = 3.7 \times 10^{-2}$. The resolution is 1024×256 in x & y , the jet beam radius is of 12 cells, so the computational box is $85.33r_b$ along x and $21.33r_b$ along y , r_b being the jet beam radius. Upper panel: $\xi = 0$ jet, & lower panel $\xi = 1$ jet. Subscripts b and a denote injected jet-beam and ambient medium quantities.

We present two shock tube tests and one 2D slab jet model. Our choice of initial conditions is within the range where the effect of ξ is significant (figure 1). The initial conditions of models for shock tube problem are presented in Table 1. In figure 2, both the models M_1 and M_2 show that the solutions depend on ξ . M_1 starts with same Θ , and in keeping with figure 1a, $\xi = 0$ is the least relativistic. While M_2 starts with same temperature and following figure 1b, $\xi = 0$ is the most relativistic fluid.

Table 1. Shock tube tests. Initial discontinuity at $x = 0.5$.

Models	ρ_L	v_{xL}	v_{tL}	p_L	ρ_R	v_{xR}	v_{tR}	p_R	ξ
M_1	1	0	0.9	1	1	0	0	10^{-5}	0.2 (points)
	"	"	"	"	"	"	"	"	1 (dashed)
	"	"	"	"	"	"	"	"	0 (solid)
M_2	1	0	0	17.182	1	0	0	1.685×10^{-5}	0
	"	"	"	3.739×10^{-2}	"	"	"	3.669×10^{-8}	0.5
	"	"	"	1.872×10^{-2}	"	"	"	1.836×10^{-8}	1

Figure 3 shows the density contours (log scale) and velocity vectors in the $x - y$ plane, the initial conditions are mentioned in the caption. The two jets are launched with the same Mach number, and $\xi = 1$ is faster and hotter than the jet described by $\xi = 0$ fluid. Therefore, we show that time dependent solutions are significantly influenced by the composition.

Acknowledgements

IC acknowledges M. Sharma, R. Joshi and A. Soam for their help in plotting the figures. DR acknowledges the support of the National Research Foundation (NRF) of Korea through grant 2007-0093860.

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